z = number of streams in a problem
$\Delta T_{\min}$ = minimum temperature difference within heat exchangers

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II. Evolutionary Generation of Networks with Various Criteria of Optimality

An evolutionary method is presented for the synthesis of heat exchanger networks. Starting from feasible solutions which preferably exhibit maximum energy recovery,* the method allows systematic promotion of desired design features such as low overall cost, suitability for starting-up procedures, observation of safety constraints, etc. Seven examples based on standard literature problems are used to illustrate the method.

SCOPE

In Part I, previous work on heat exchanger network synthesis was discussed and a new synthesis method proposed. It allows systematic generation of networks which achieve maximum energy recovery.

However, attempts to obtain useful interaction between the systematic approach of a formalized method and the imaginative, heuristic based intelligence of the experienced designer may not be automatically successful. As with other systematic methods, variations of design constraints such as $\Delta T_{\min}$ (that is, the minimum temperature difference to be permitted within an exchanger), or of costing parameters, are possible, but whether or not such variations will produce desired differences in the final networks cannot be easily predicted. Once the design constraints are defined and the criteria of optimality approximated in a feasible quantitative manner, an automatic algorithm will base decisions on marginal numerical differences which may not be very relevant from points of view such as safety, control, reliability, etc.

Until recently, the literature dealing with the synthesis of heat exchanger networks offered little advice in this matter. The identification of near optimum cost structures for a given problem was usually chosen as the objective. Rathore and Powers (1975), however, observed that the costs of networks showing a high degree of energy recovery are near optimal as well as quite insensitive to significant changes in network topology. They suggested the following strategy: identification of a number of networks featuring high degrees of energy recovery, and final selection of one of these networks using whatever combination of criteria seems relevant.

Apart from enabling the user to consider aspects other than costs, such an approach has the merit that not all feasible structures have to be enumerated. This helps to overcome the problem of dimensionality which is usually very large in the synthesis of heat exchanger networks.

There are, however, two basic disadvantages in the approach. Firstly, the computational effort required per final candidate structure identified may not be significantly reduced. Secondly, some suitable candidates might not be found because they narrowly fail to meet design constraints (such as $\Delta T_{\min}$) or the problem specifications (such as a target temperature for a stream).

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* The synthesis of such solutions has been discussed in Part I (see Linnhoff and Flower, 1978).

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A synthesis technique of any realism should not only be capable of identifying optimal networks for a strictly defined problem but should also allow the user to recognize chances where they exist of slightly relaxing problem constraints to gain some advantage.

Shah and Westerberg (1975) and Nishida et al. (1977) suggested the use of evolutionary rules for the synthesis of low cost networks. This approach to the synthesis problem is attractive owing to the ease with which interactive computer-user participation is made possible. However, an approach based on heuristic rules will always require the user to make some decisions. This is especially true when the objectives include matters such as safety, operability, etc., which are less well interpreted in terms of network structures than overall cost.

In the present paper, a synthesis method is presented which is suitable for hand implementation or for use as an interactive computer program. It enables the user to use evolutionary rules such as suggested by previous workers with great ease as well as to gain considerable insight into the consequences of future synthesis steps. This allows heuristic rules to be replaced when necessary by other evolved strategies without increased effort. Thus, desired network features can be actively promoted during the synthesis, rather than passively emerging from a limited number of trial solutions.

The method uses a systematic representation of heat exchanger networks, in which exchangers and streams form a grid. In such a grid, each exchanger may be interpreted as consisting of a heater and a cooler. Heaters and coolers, however, may be shifted through the grid according to simple feasibility rules. These rules help to determine what the consequent effects of a shift on the network would be. Starting from a given structure, any other structure may be obtained. This includes cyclic networks (that is, networks in which the same two streams are matched against each other more than once) as well as topologies which incorporate parallel stream splitting.

CONCLUSIONS AND SIGNIFICANCE

The evolutionary development method (ED method) enables the user to obtain rapid insight into beneficial, or detrimental, effects of potential synthesis steps in the design of heat exchanger networks. Thus, the formulation of strategies aimed at achieving given design goals is greatly helped. This eliminates, by and large, the need to accumulate many different solutions to a given problem before checking whether any display a desired combination of features in aspects such as cost, operability, safety, etc. Instead, the desired features can be methodically introduced in a sequence of evolutionary steps, starting from any feasible structure. Preferably, but not necessarily, the structure from which one starts should exhibit maximum energy recovery. Such structures can always be found by means of the TI method described in Part I.

Design strategies which would help to develop networks of low overall cost have been found. Some of them are related to heuristic rules for evolutionary network development given by previous workers; others are new. When implemented by means of the ED method and applied to seven examples from the literature, they produce new optimum cost solutions which are improvements on those previously described in five cases. In the remaining two cases (that is, 4*SP*1 and 6*SP*1, see text), optimum solutions found by previous workers were confirmed. In some cases, the TI method was used to provide initial maximum energy recovery networks, while in other cases, the networks of previous workers were used as starting points.

Perhaps more important, though, solutions were synthesized for several problems which appear to be better candidates from a practical point of view than the respective optimum cost networks. Most of these solutions combine near optimum cost with a high degree of flexibility in terms of choice of certain design and/or operating parameters. One particular network proposed (for 10*SP*1, see text) narrowly fails to meet the problem specifications but compensates for this disadvantage by exhibiting an attractively simple structure.

Further, criteria were identified which are applicable to the design of heat exchanger networks in general and which seem to be of great importance. It has been previously noted that investment costs for plant are mainly influenced by the number of heaters, coolers and exchangers used (for example, Nishida et al., 1977), but, apart from the work of Hohmann (1971), there is little discussion of the minimum number of units required in a given problem. Apart from costs, points of concern such as flexibility of design and the potential benefits of parallel stream splitting are shown to be closely related to the number of pieces of equipment used. Three main points emerged: (1) stream splitting may be employed to reduce the number of exchangers where this cannot be done in any other way, (2) sizes of exchangers may be varied if more pieces of equipment are used than the minimum number strictly necessary, (3) stream splitting may allow variation of exchanger sizes without increasing the number of exchangers.

Intelligent choice of the number and sizes of those pieces of equipment which are not totally constrained and a flexible attitude towards design constraints, combined with the ability to direct the synthesis towards a desired topology, should make satisfactory solution of realistic design problems possible by the ED method.

PROBLEM STATEMENT

As stated in Part I, most of the recent work in heat exchanger design has concentrated on the solution of identical types of problems, and a brief description has been given there.

In the present paper, solutions will be sought to seven problems from the literature, but apart from annual costs, aspects will be considered such as the degree of interconnection of streams (which would influence controllability), flexibility in design, possible safety constraints, etc. The design and costing parameters to be used are those of Pho and Lapidus (1973) to ensure comparability with previous work.

THE CONCEPT OF FREEDOM AND THE ED METHOD

In Figure 1*a*, the optimum solution found by Masso and Rudd (1969) for problem 7*SP*1 is shown. The method of graphically presenting the network is the

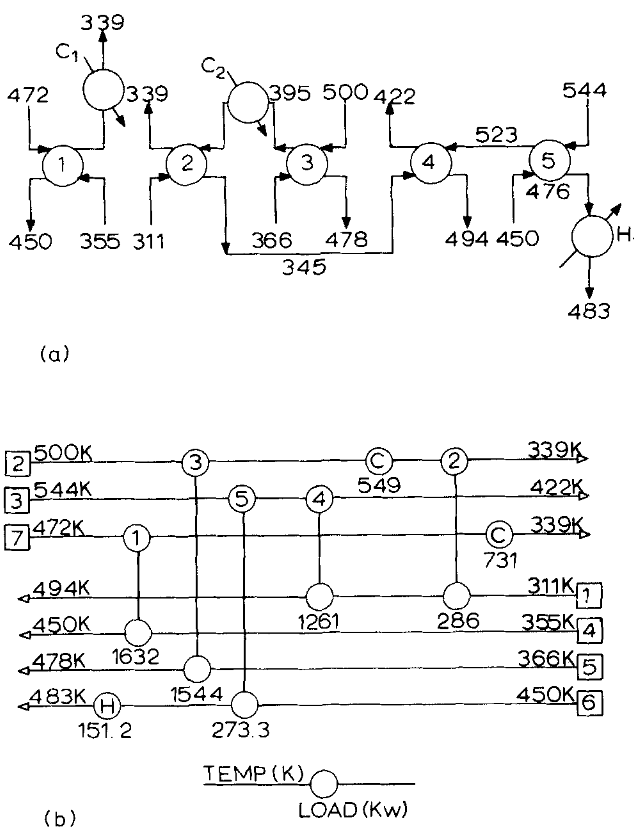

Fig. 1. 7SP1 as solved by Masso and Rudd. (a) Conventional drawing. (b) Grid representation.

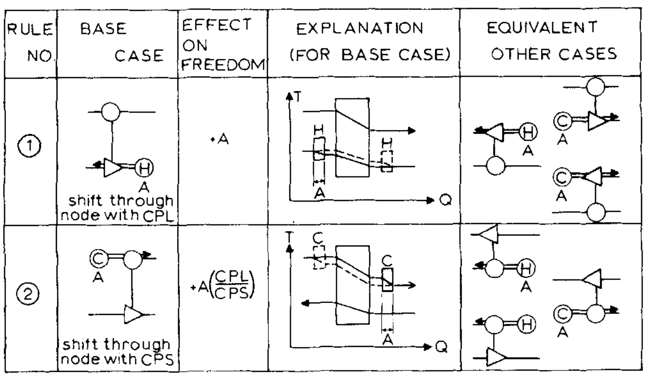

Fig. 2. Shifts along streams of heaters and coolers through exchangers.

same as adopted by Masso and Rudd themselves. In Figure 1*b*, the network is shown by means of a method introduced in Part I. The diagram represents a grid, made up of streams and exchangers. Hot streams run from the left to the right at the top, and cold streams run from the right to the left. Beneath the symbols for heaters, coolers, and exchangers, heat loads are noted in appropriate units (kilowatts for all examples introduced in this paper). Temperatures may be shown against each stream and so may heat capacity flow rates. The exchangers may be numbered at one of their nodes.

The explanation of the synthesis method is conveniently based on this diagram which can represent any feasible network. In turn, particular features of the diagram can be justified by an explanation of the method.

The Definition of Freedom

In Equation (1), a parameter is introduced which is called the "freedom" of a heat exchanger:

$$F = CPL(\Delta TS - \Delta T_{min}) \quad (1)$$

This parameter has the same physical dimension as a heat load. The purpose of introducing this parameter is to ease methodical investigation of the effects of changes in the positions of exchangers. The effects of such changes are normally discussed in terms of the changes in temperatures and temperature differences which occur in the exchanger, since these changes affect the feasibility of the revised network in a quite familiar and fundamental way. However, relating the size of shifting heat loads and the temperature differences in this way requires continual reference to the heat capacity flow rates of the individual streams. It is more convenient to describe the effects on the heaters and coolers in the network in terms of the temperature changes of the streams passing through the heaters and coolers. In this way, the effects of shifts are more easily understood by comparing temperature differences in exchangers with temperature drops across heater and cooler loads.

If the heat capacity flow rates are constant over reasonable ranges of temperature, then the temperature drop across a load will be independent of its position on the stream. Thus, shifting a load through an adjacent exchanger will alter the smallest temperature difference in the exchanger by the fixed value of the temperature drop across the load.

In a similar way, the temperature differences between the source of utility heat or cold and the process stream in a heater or cooler will change. In many applications, the temperatures of the utilities will be so different from those of any feasible process stream that the possibility of the heater or cooler showing infeasible temperature differences can be ignored. In other cases, the feasibility of the heater or cooler can be investigated in a way exactly similar to that described below for an exchanger.

Alternatively and preferably when more complex changes are considered, the scope for shifting can be interpreted in terms of the maximum load which can be shifted through the exchanger before the point of infeasibility is reached. The maximum load which can be shifted through an exchanger in a simple case is equal to the freedom as defined by Equation (1). Shifts of smaller loads will merely reduce the freedom of the exchanger by the size of the load.

While the use of temperature differences has the advantage of familiarity, the concept of freedom has equally important advantages in complex situations, even when the enthalpy-temperature curve shows marked nonlinearity (Linnhoff, 1978).

Basic Elements of Synthesis Method

Figure 2 summarizes the effects that simple shifts of heaters and coolers have on an exchanger's freedom; shifts according to rule No. 1 (that is, along the stream with the larger Cp) will alter the exchanger's freedom by the heat load of the cooler (or heater) which is shifted. The effect of shifts according to rule No. 2 (that is, along the stream with the smaller Cp) is greater by the factor CPL/CPS (where CPS is the smaller of $Cp_{\text{hot stream}}$ and $Cp_{\text{cold stream}}$). To make it easily recognizable which one of the two streams matched in a particular exchanger has the larger Cp, the following conventions have been adopted:

1. To situate an exchanger's reference number in the node on the stream with the smaller Cp.

2. To replace the node on the stream with the larger Cp by a triangle as a qualitative indication of the temperature profile within the exchanger. Thus, it will always point, like an arrow, towards that side of the exchanger at which the smallest actual temperature difference is

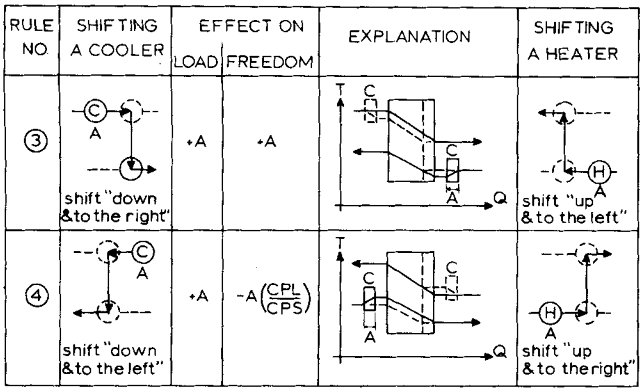

Fig. 3. Shifts from one stream to another of heaters and coolers through exchangers.

found. In the case of equal *Cps* for both streams, two circles may be retained. For exchangers in which the freedom is zero (that is, ΔT_{min} is reached), the triangle or one of the circles (in the case of equal *Cps*) may be blacked out.

In Figure 3, two more rules are given. They are concerned with cases in which the heater (or cooler) in question is shifted from one stream to another. Such shifts not only affect the freedom of the exchanger but also affect its load. On the other hand, the effects are independent of the heat capacity flow rates so the exchangers are not shown in full detail in Figure 3.

It will be appreciated that the direction of each shift shown in Figures 2 and 3 may be reversed and that coolers may be shifted instead of heaters and vice versa. The effects on load and freedom of the exchanger concerned would be equal in magnitude to the effects shown in Figures 2 and 3 but opposite in sign. Furthermore, those combinations of rules No. 1 to No. 4 are shown in Figure 4 which have been found to be of practical value. Each of these combinations, again, could be applied with reversed direction and reversed sign.

Applying the Method to 7SP1

In Figure 5*a*, the network presented in Figure 1*b* is shown again with the new symbols. The freedoms of the exchangers are also shown (vertically, in the space between hot and cold streams). When choosing which evolutionary synthesis step would be most beneficial to the network, one should always question the necessity of a small heater being present as well as coolers. Unless there is a strict thermodynamic reason for such an arrangement (for example, a cold stream becoming hotter than any hot stream), utility resources are wasted on grounds of both steam consumption and a corresponding requirement for additional cooling water.

To eliminate the heater, however, would only be possible if it could be shifted to a point in the network next to a cooler (so that a fraction of the cooler could cancel the heater). Consequently, the mobility of the heater in the network has to be examined. This can be done by simply comparing its heat load (that is, 75.58 kW) with the freedoms of the exchangers in the network (if appropriate, after consideration of the relevant factor *CPL/CPS*). It quickly becomes clear that the heater could be shifted through any of the exchangers except No. 2. The freedom of exchanger No. 2 (that is, 32.24 kW) is too small. On the other hand, the cooler on stream No. 2 can be shifted through exchanger No. 2, thereby increasing the exchanger's freedom (see dotted line in Figure 5*a*). It is then possible to shift the heater through the network along the dashed line, and the structure shown in Figure 5*b* results. The shift through exchanger No. 5 took place according to rule No. 4, the one through exchanger No. 4 according to rule No. 3, and rule No. 4 was used, once again, for the shift through exchanger No. 2. The freedoms in structure 5*b* may either be calculated by means of these rules based on the values shown in structure 5*a* or they may be evaluated from temperatures and heat loads, as given in Figure 5*b*, by means of Equation (1).

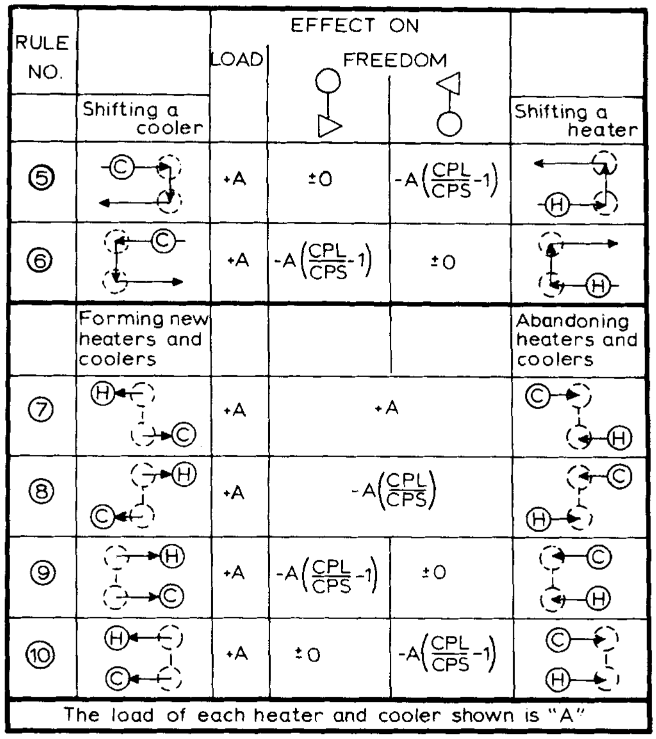

Fig. 4. Further rules for shifting and merging.

With an annual cost of only $30 172, structure 5b is over 12% cheaper than structure 5*a*. It is, in fact, the cheapest solution presented so far to the problem. Apart from Masso and Rudd, only Pho and Lapidus

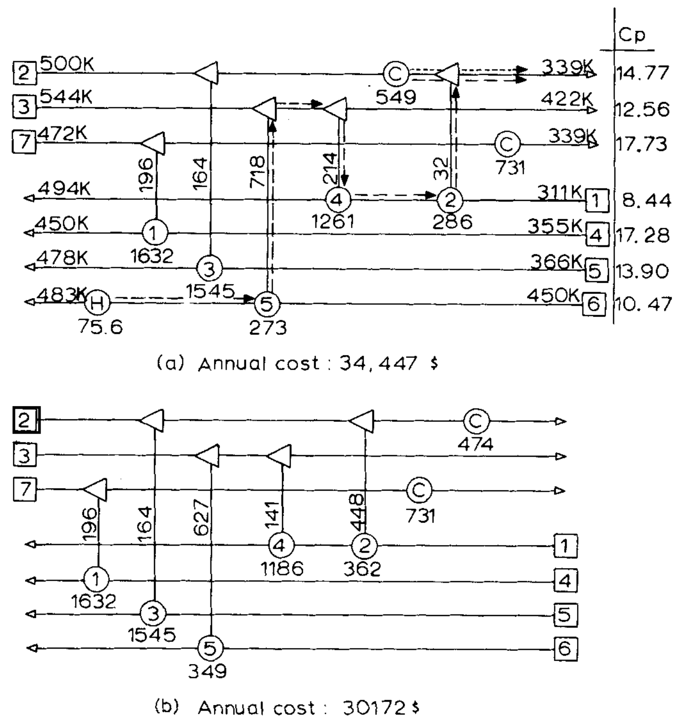

Fig. 5. Evolutionary improvement of solution for 7SP1. (a) Solution as shown in Fig. 1. (b) Optimum cost solution.

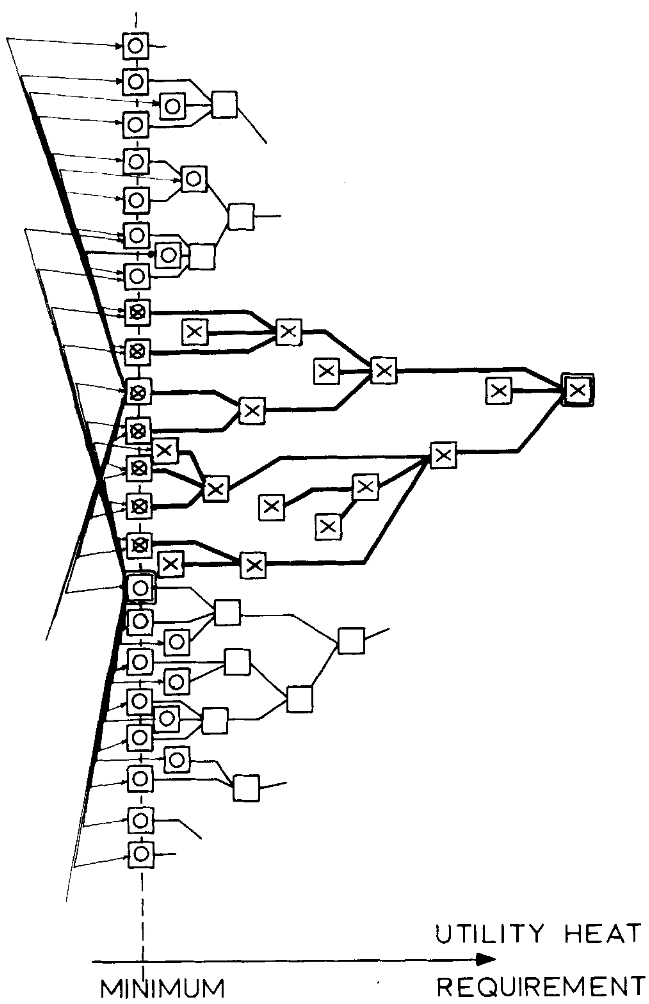

Fig. 6. Section of a simplified synthesis tree.

(1973) and Ponton and Donaldson (1974) have solved 7SP1. Pho and Lapidus claimed an annual cost of $30 433 for their solution owing to an error in cooling water costs, as noted by Nishida et al. (1977). The true cost for their solution is approx. $31 700/yr. Thus, the network shown in Figure 5*b* represents some worthwhile improvement. Ponton and Donaldson did not present a network but reported finding one (by means of their H/2H search algorithm) costing $30 172/yr, too. It seems quite certain that they found the same structure as presented in Figure 5. A short search with the rules as presented in Figures 2, 3, and 4 shows that there is no other solution for 7SP1 with such a low cost.

Evolutionary Development Instead of Depth First

In Figure 6, a conventional heat exchanger network synthesis tree is presented symbolically. Each square represents a particular structure. The position of a square in the diagram indicates the energy performance achieved by the structure which is represented. Networks operating with minimum requirement for process utility heat are situated on the far left. In branch and bound and in tree searching algorithms, such synthesis trees are evolved by consistently adding further matches to existing structures, so that new structures result from the existing ones. Thus, all feasible networks can, theoretically, be derived from primary matches. Practically, however, there are too many enumerations required even for small problems.

To overcome this problem, Rathore and Powers recommended giving emphasis to the rapid evaluation of several, but not all, structures with a high degree of energy recovery "For a large industrial problem . . . . developing any of the decision nodes containing a primary match will probably lead to a number of networks which are close to the cost of the optimal one." This approach (that is, depth first) is indicated in Figure 6 by means of crosses; starting off from primary match [X], a certain number of optimum and near optimum energy networks are found. Since overall costs are heavily dominated by utility costs, these networks will all be ". . . close to the cost of the optimal one" (see above).

The evolutionary approach to synthesis favored here is indicated in Figure 6 by means of circles; starting off from optimum energy structure [O], other structures situated either on or near the optimum energy line may rapidly be evolved, no matter how many steps back in the tree the common origin lies. From each structure found in this way, further structures may be evolved. (This is indicated for only one case in Figure 6.)

In recent studies, Shah and Westerberg (1975) and Nishida et al. (1977) suggested a similar approach. In both these studies, however, a limited number of evolutionary rules were given which would help to produce a limited number of neighboring structures starting from a feasible base. The feasibility rules presented in this work (see Figures 2, 3, and 4), by contrast, are based on generally applicable thermodynamic principles and allow the user to find any feasible solution starting from any other feasible structure. Thus, while rules such as those suggested by Shah and Westerberg and by Nishida et al. could be called strategic rules, the rules presented in this paper could be called tactical rules, enabling any strategic policy to be implemented. The identification of such tactical feasibility rules seems appropriate, since strategic policies might be more easily formulated if available tactical options are more clearly appreciated. The networks used as starting structures do not necessarily have to lie on the optimum energy line (see, for example, Figure 5), but experience has shown that it is very convenient if they do. The problem of identifying an optimum energy starting point in the first place can always be solved by means of the TI method introduced in Part I.

It is evident that the ED method may be capable of identifying a greater number of promising network structures with less computational effort than the depth first strategy. What may be an even more valuable asset of the new method, however, is the fact that particular network characteristics can, within certain limits, be deliberately developed or suppressed. Thus, it is not necessary to carry out an exhaustive search through a defined part of the solution space (in the hope that one of the solutions obtained would turn out suitable in the light of whatever combination of criteria seems relevant) since it is possible to concentrate one's effort on the development of only those structures which appear to lead to suitable final networks. The approach taken in Figure 5 may serve as an example. The goal was to eliminate the heater, if possible, and the strategy adopted was simply to shift the heater through the network along that particular path which would bring it next to a cooler. The ED method, based on the concept of freedom as presented here, makes it possible to consistently pursue suitable strategies in particular situations. To substantiate these claims, further examples will now be solved.

4SP2, CYCLIC NETWORKS AND STREAM SPLITTING

Ponton and Donaldson (1974) suggested that cyclic arrangements of heat exchangers had to be considered to solve this problem satisfactorily. They presented a

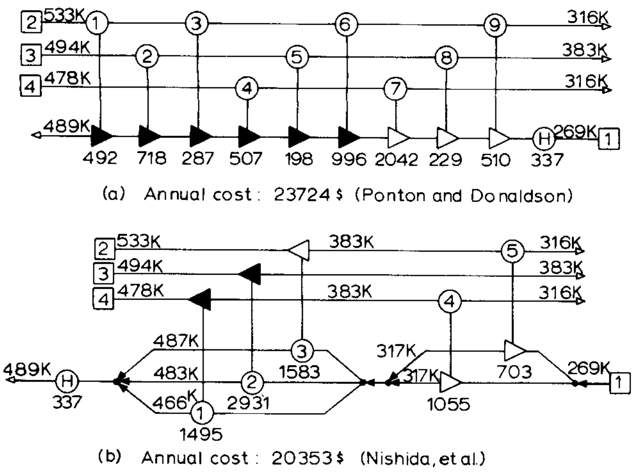

Fig. 7. Previous solutions for 4SP2. (a) Ponton's and Donaldson's solution. (b) Nishida, et al.'s solution.

SN	Hot streams (2), (3), (4): T (K)	Cold stream (1): T (K)	Maximum permissible subnetwork heater load (kW)	Maximum permissible subnetwork cooler load (kW)
SN(1)	533.2 – 499.8	– 488.7	337	688.7
SN(2)	499.8 – 494.3	488.7 – 483.2	688.7	542.2
SN(3)	494.3 – 477.5	483.2 – 466.5	542.2	542.2
SN(4)	477.5 – 383.2	466.5 – 372.0	542.2	2037.0
SN(5)	383.2 – 316.5	372.0 – 305.4	2037.0	1334.0
SN(6)	316.5 –	305.4 – 269.3	1334.0	0.0
	Cp = 10.55 (2); Cp = 26.38 (3); Cp = 15.83 (4) (Cp in kW/K)	Cp = 36.93		

Fig. 8. Problem table for 4SP2.

solution featuring four different matches between the same two streams and costing \$23 724/yr (see Figure 7*a*) and compared this to the annual cost of acyclic solutions which would be about three times as high. Nishida et al. (1977), however, showed that a further significant saving was possible if parallel splitting of stream No. 1 was considered and presented a solution costing \$20 353/yr (see Figure 7*b*). New solutions will now be sought by means of the TI method as described in Part I (to generate an optimal energy network) and the ED method (to produce satisfactory final solutions). First, solutions which do not incorporate stream splitting will be produced, and, afterwards, stream splitting will be considered.

A Cyclic Solution for 4SP2

In Figure 8, the problem table for 4SP2 is shown. (For a detailed explanation of the problem table, see Part I) From this table, the data for the problem are easily identified as well as the maximum permissible loads on heaters and coolers within each subnetwork. These loads must not be exceeded if the final network is to give maximum energy recovery. In Figure 9*a*, an array of subnetworks is shown which complies with this condition. In Figure 9*b*, four pairs of heaters and coolers have, according to the principles discussed in Part I, been arranged ready for the formation of new exchangers. In Figure 9*c*, these four matches have been formed, and neighboring units have been merged. Structure 9*c* is optimal from an energy recovery point of view.

At this point, the ED method can be employed to identify further solutions in the manner described in Figure 6. In the present case, an attempt was made to reduce the number of exchangers in the network as far as possible. This should lead to simpler topologies.

Looking for a chance to eliminate superfluous exchangers, one easily finds, by comparison of heat loads and freedoms, that exchanger No. 3 may be merged partly into exchanger No. 7 and partly into exchanger No. 10 (see dashed line in Figure 9*c*); match No. 3 may be visualized as consisting of heaters and coolers, and the freedoms of exchangers No. 4, 5, and 9 are big enough to allow a heater of the required size (for example, 128.95 kW) to be shifted on stream No. 1 towards the cold end (rule No 1). Also, the freedom of exchanger No. 7 is big enough to allow a merging of a heater and a cooler (loads: 35.17 kW) according to rule No. 9. The other two necessary operations (shift through match No. 8 according to rule No. 1 and merging into match No. 10 according to rule No. 10) do not decrease freedom, that is, must be possible.

Once match No. 3 has been eliminated, matches No. 8 and No. 4 become neighbors and can be merged. A network results which consists of six exchangers and one heater (Figure 9*d*). Its annual cost is \$21 654. This compares favorably with the cost of the solution found by Ponton and Donaldson which is \$23 724/yr.

Further reductions in the number of exchangers have not been found. Six exchangers appear to be the minimum number required if stream splitting is not considered. There are, however, two other topologies featuring six exchangers, and they, too, are easily identified by means of simple shifting operations such as demonstrated in Figure 9*c*. The structures in question are shown elsewhere (Linnhoff, 1978).

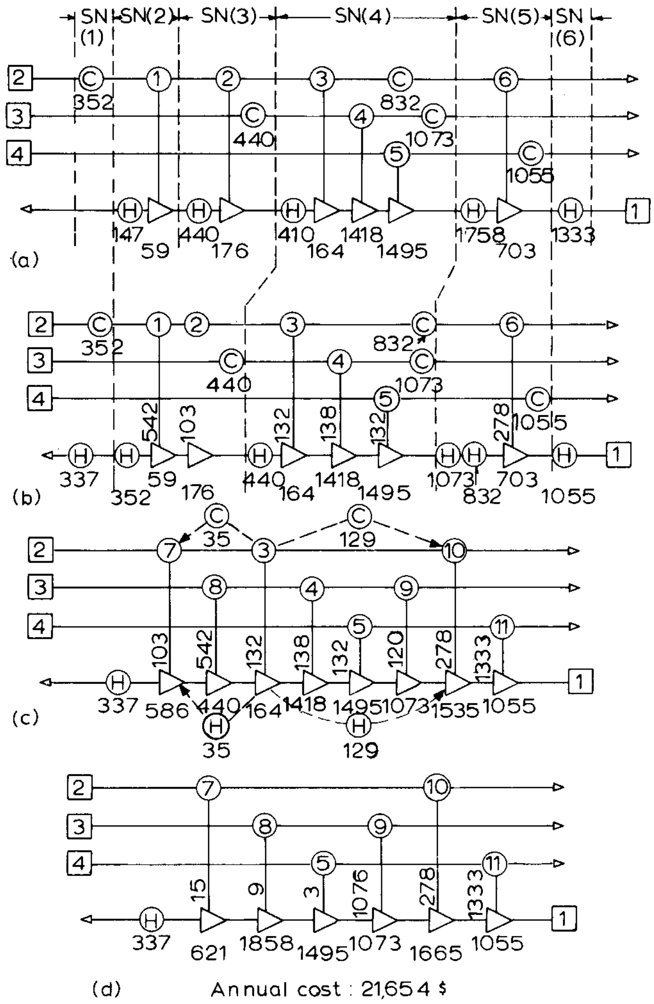

Fig. 9. Complete synthesis for 4SP2. (a) Array of subnetworks. (b) Heaters and coolers ready for formation of new exchangers. (c) Eliminating match No. 3. (d) Final solution without stream splitting.

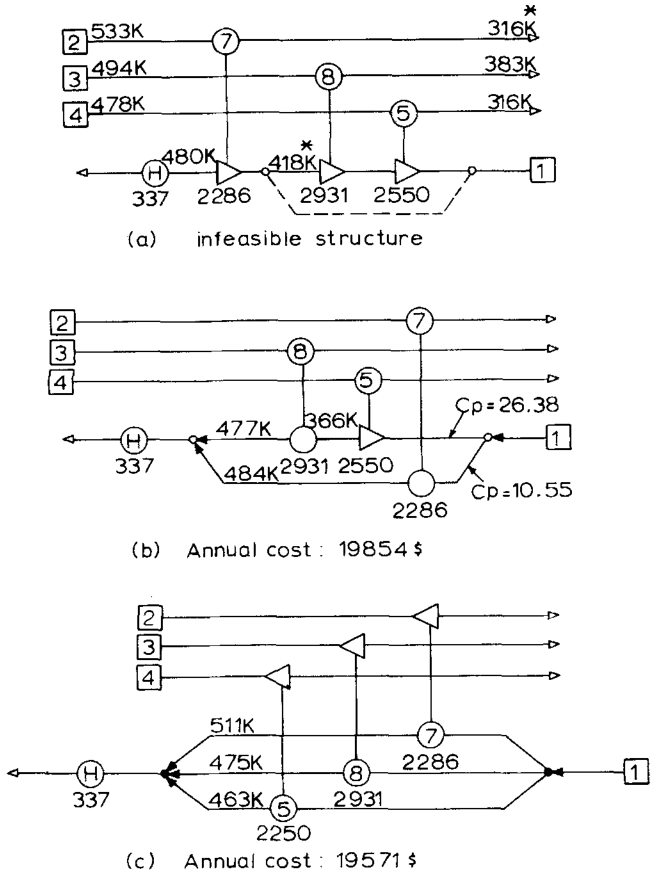

Fig. 10. Split stream solutions for 4SP2.

Solutions for 4SP2 Incorporating Stream Splitting

As Nishida et al. (1977) pointed out, costs of networks are likely to decrease with decreasing number of exchangers, and since their solution for 4SP2 incorporates only five exchangers, as compared to the six exchangers required, for example, in structure 9*d*, it is significantly cheaper.

On the other hand, it is quite clear that with three hot streams and one cold stream, there need only be three distinct heat exchange tasks between the process streams. Attempts to find structures which use three exchangers proved successful.

In Figure 10*a*, an infeasible structure is shown which has been obtained from structure 9*d* by simply merging matches No. 10 into No. 7, No. 9 into No. 8, and No. 11 into No. 5. The resulting matches No. 8 and No. 5 are feasible, but in match No. 7, the temperature difference at the cold end has become infeasible, however, this problem can easily be solved by splitting stream No. 1 in the fashion indicated in Figure 10. If a bypass is introduced around matches No. 8 and No. 5 (see dashed line in Figure 10*a*), match No. 7 may be shifted into the bypass which will ensure that the temperature difference at its cold end becomes feasible. To ensure that its temperature difference at the hot end does not, in turn, become too small, the heat capacity flow rate chosen for the bypass must not be chosen below a certain threshold value. Since

$$Cp_{\text{bypass}} \geqq \frac{\text{Head load in bypass}}{\text{Maximum temperature rise in bypass}} \quad (2)$$

the minimum value is found to be 9.041 kW/°K. Similarly, any figure below Cp = 25.621 kW/°K for the remaining branch of stream No. 1 would lead to a violation of the $\Delta T_{\min}$ constraint in match No. 8. Thus, the total heat capacity flow rate of both branches of stream No. 1 in Figure 10*b* must not lie below 34.663 kW/°K. This condition is compatible with the actual heat capacity flow rate of stream No. 1 (that is, 36.926 kW/°K) and leaves some room for further refinement of exchanger areas.

The two values adopted in Figure 10*b* (that is, 10.55 and 26.375 kW/°K) have been chosen purely on grounds of computational convenience. The resulting network is, with \$19 571/yr, about \$500/yr cheaper than Nishida et al.'s (1977) solution.

To reduce the costs further, stream No. 1 can be split twice, which leads to yet another topology with only three exchangers, see Figure 10*c*. The way in which structure 10*c* may be found from either structure 10*a* or 10*b* is entirely analogous to the way in which structure 10*b* was evolved from 10*a*. The cost of structure 10*c* is \$19 567/yr. A short search makes it clear that structures 10*b* and 10*c* are the only two topologies which exist for 4SP2 with only three exchangers. However, both topologies allow considerable variation of the heat capacity flow rates in the different branches of stream No. 1, and the cost for both solutions could still be slightly improved by optimizing the network with respect to these parameters.

According to the cost parameters as given by Pho and Lapidus (1973), structure 10*c* is about 4% cheaper than Nishida et al.'s solution. In practical terms, however, the costs of additional valve gear and pipe fittings would probably be quite sensitive to stream splitting. On these grounds, it is possible that structure 10*b* might be the most attractive solution for the problem.

4SP1, THE MINIMUM NUMBER OF UNITS AND FLEXIBILITY IN DESIGN

Problem 4SP1 was first presented by Lee et al. (1970) and has been solved several times in the literature. Siirola (1974) presented the optimum solution. The problem will now be solved by means of the TI and the ED methods. Firstly, this allows a good comparison of these methods with other methods. Secondly, it pro-

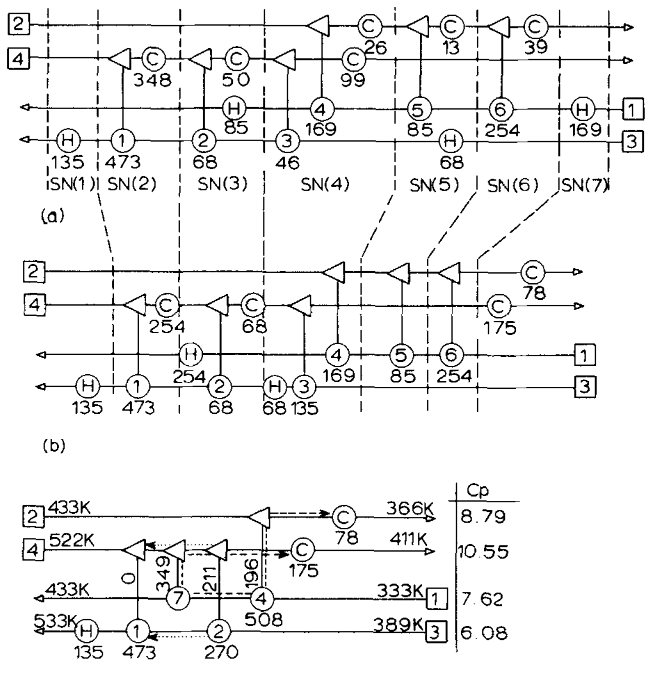

Fig. 11. Synthesis of energy optimal network for 4SP1. (a) Array of subnetworks. (b) Subnetworks after shifting of heaters and coolers. (c) Preliminary network (with optimum energy recovery).

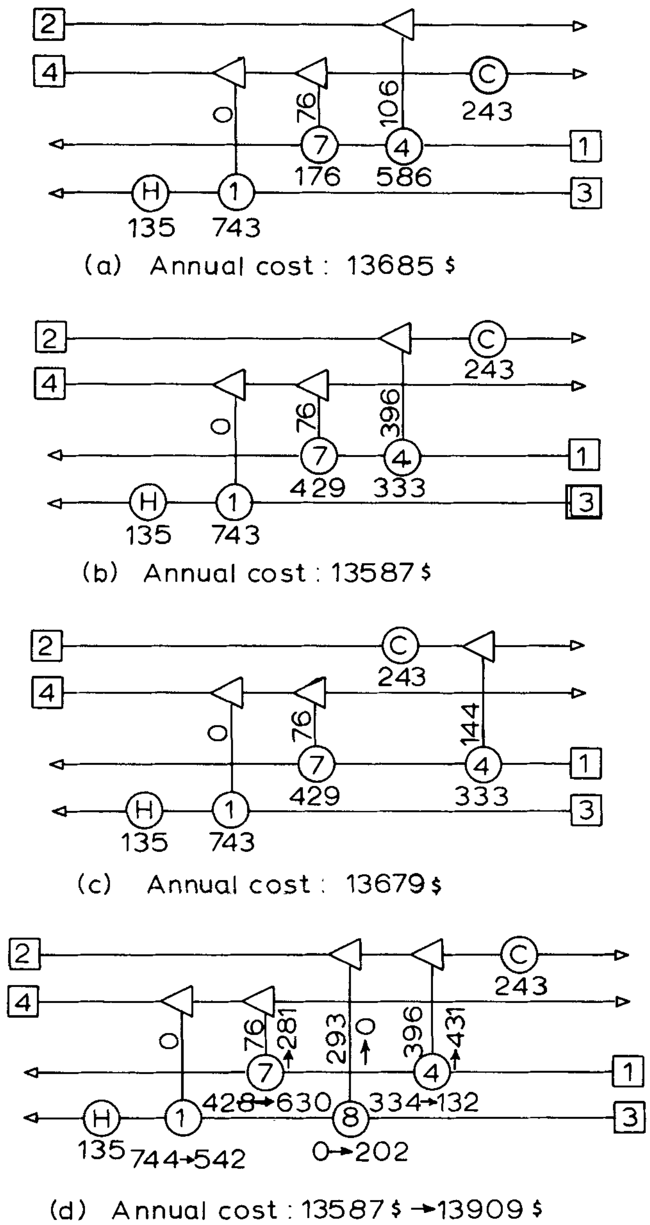

Fig. 12. Four final solutions for 4SP1.

vides a suitable basis for the discussion of the importance of the number of units on the flexibility of a network.

Solving 4SP1

In Figure 11*a*, an array of subnetworks is presented for 4SP1. In Figure 11*b*, two pairs of heaters and coolers are ready for merging, and in Figure 11*c*, the new exchangers have been formed, and neighboring units have been merged. The resulting network shows maximum energy recovery. The problem data for 4SP1 can easily be deduced from Figure 11*c*.

To reduce, again, the number of exchangers as far as possible, there are two obvious possibilities in Figure 11*c*. Firstly, exchanger No. 2 may be shifted through exchanger No. 7 (the freedom of match No. 7 is big enough) so that it can be merged with match No. 1. Secondly, the two coolers may be merged into one unit, either on stream No. 2 or on stream No. 4. These two changes are indicated in Figure 11*c* by means of dotted and dashed lines, respectively. Two networks emerge (see Figures 12*a* and 12*b*) with three exchangers, one cooler, and one heater, that is, five pieces of equipment. From the freedoms and heat loads in structure 12*b*, it is evident that the cooler on stream No. 2 may also be situated on the hot side of match No. 4. The third (and last) structure with no more than five pieces of equipment is found, see Figure 12*c*.

The Minimum Number of Units

Rathore and Powers (1975) concluded that the three structures *a*, *b*, and *c* in Figure 12 are the three cheapest solutions. This is understandable, since they are the only three structures found with no more than five units; as mentioned above, networks involving fewer but larger units tend to cost less. Thus, it would be a worthwhile approach, in general, to try and identify the minimum number of units required to solve a problem and to try and generate networks which use this number.

Hohmann (1971) proposed that a lower bound on the number of units be defined by the following rule:

> The minimum number of units is nearly always one less than the number of streams and services required in the problem (3).

Figure 13 illustrates for the case of 4SP1 that the minimum number is five. The reservation nearly always has to be made since it is not impossible that heat loads of a hot and a cold stream are equal to each other (or to the total load on coolers or heaters), or residuals turn out to be equal to each other or to original heat loads. In any one of these situations, the minimum number of units is one less than that suggested in expression (3).

However, all these considerations are just based on the compatibility of heat loads and ignore temperatures. Evidently, both temperatures and heat capacity flow rates define whether or not a problem can be solved with the minimum number of units. Particular combinations of data might make it necessary to resort to cyclic topologies or to parallel stream splitting, and, contrary to the comments made by Hohmann (1971), the minimum number of units may not always be achievable.

Flexible Designs

As is self-evident from Figure 13, solutions using the minimum number of units show fixed exchanger loads; exchanger sizes can only be varied, to a very limited

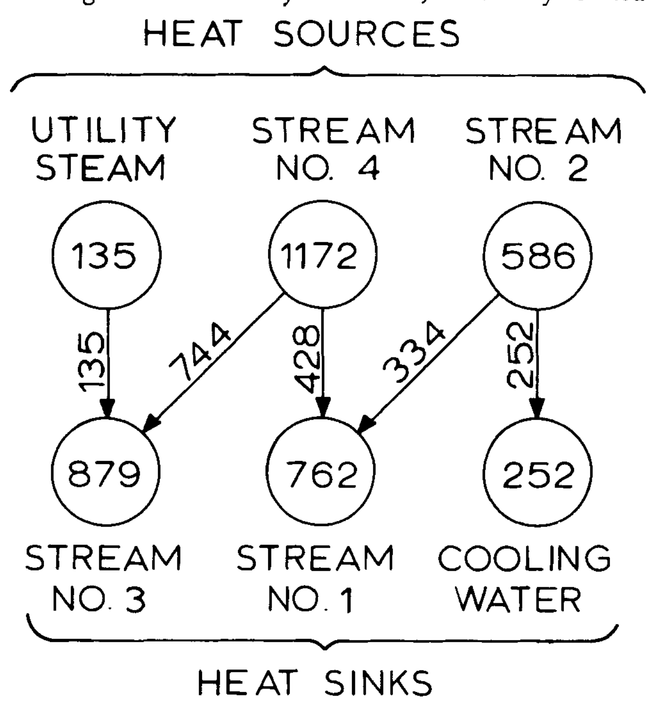

Fig. 13. Finding the minimum number of units. All figures apply for 4SP1. The figures for "utility steam" and "cooling water" are evaluated by means of the problem table and the assumption of maximum energy recovery.

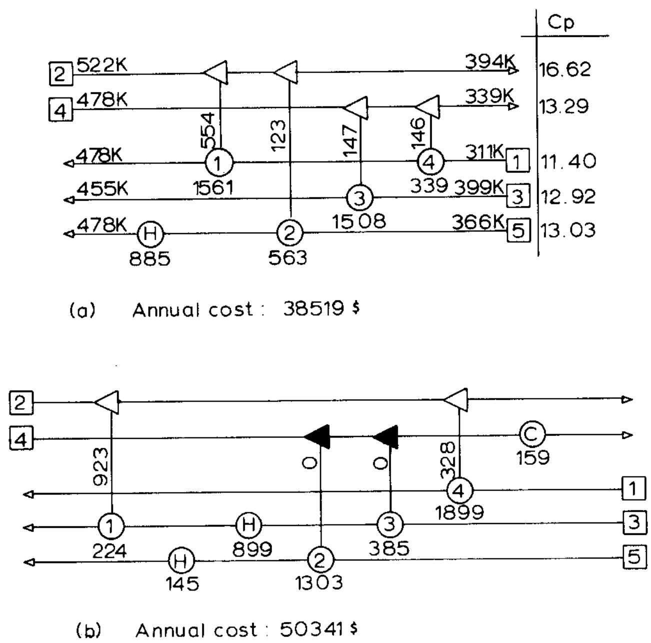

Fig. 14. Two solutions for 5SP1. (a) Near-optimum-cost network. (b) Solution without a match between streams No. 4 and No. 1.

extent, by altering the sequence of the matches (compare, for example, structures 12*b* and 12*c*).

In Figure 12*d*, a solution is shown for 4SP1 with six pieces of equipment rather than five. Many such solutions can be made up by introducing additional matches, one at a time, into any of the three solutions with only five units. (Rathore and Powers, for example, have shown three other networks of this type.) The interesting common feature of all such networks is the fact that they allow variation of certain design and/or operating parameters within certain ranges. This has been indicated in Figure 12*d* by noting the full possible range for heat loads, as well as for freedoms. Rathore and Powers quote solutions of this type with fixed values found at one extreme end of the feasible range. Similarly, other previous workers have often presented potentially flexible solutions, without explicitly noting flexibility. Dealing with problems like control behavior, step changes in available size of equipment, etc., such flexibility will be a valuable asset of a design. With the rules for shifting and merging as introduced in Figures 2, 3, and 4, the range of flexibility is easily identified; in Figure 12*d*, for example, the range is limited by the heat load of match No. 8 at one end and by the freedom of match No. 8 at the other end. As far as costs are concerned, such flexible designs can be very attractive, too; structure 12*d* has been derived from the overall cheapest solution (that is, structure 12*b*), and it can be cheaper than either of the other two networks with the minimum number of units.

5SP1, AVOIDING UNDESIRABLE MATCHES

Optimum Cost Solutions

First presented by Masso and Rudd (1969), 5SP1 has been solved by many workers who all proposed the same best solution, costing $38 263/yr (see, for example, Pho and Lapidus, 1973). An application of the TI and ED methods to the problem confirmed this solution and revealed four others, costing between $38 278 and $38 550/yr (see Linnhoff, 1978). One of these networks is shown in Figure 14*a*. Each of these five solutions features the minimum number of units [that is, five, according to expression (3)], and a great variety of flexible solutions, featuring six pieces of equipment, may be made up in the way demonstrated in Figure 12*d*. All of these would, in practical terms, be considered optimal. Nishida et al. (1977), who considered stream splitting, presented a new optimum cost solution costing $38 219/yr.* This solution will be shown later to be closely related to the structure shown in Figure 14*a*, and an examination of this relationship will lead to an improvement on Nishida, et al.'s solution.

Assuming a Safety Constraint

In Figure 14*b*, however, a solution is shown to 5SP1 which costs $50 341/yr and consists of seven pieces of equipment. The interesting feature of this solution is the fact that it does not incorporate a match between the two streams No. 4 and No. 1. Considering the hot stream target temperatures in the problem as well as the cold stream supply temperatures, it becomes clear that the installation of a cooler cannot be avoided unless streams No. 4 and No. 1 are matched against each other at the cold end. Accordingly, all near optimum cost solutions mentioned above incorporate such a match. If, for safety reasons, this match cannot be allowed, the solution of the problem becomes somewhat more difficult. One possible approach would be to use the TI method, avoiding the use of the forbidden type of match already in the subnetworks. Another approach would be to take any of the original low cost solutions, replace the forbidden match by a cooler and a heater, and try to form new exchangers from this heater and cooler (after shifting and merging) to as large an extent as possible. A variety of solutions will be obtained. The one shown in Figure 14*b* is not proposed as the definite cost optimum, but it has been chosen because it does not feature exchangers of unrealistic size as do other solutions which are marginally cheaper.

Relevance of Costing Equations

Before we discuss solutions to the other problems, it is worth pointing out an important shortcoming of the usual 0.6 power of area costing equation for heat transfer area as given by Pho and Lapidus. It is well known that, according to this equation, total surface area costs least if it is distributed over as few separate exchangers as possible, see, for example, Hohmann (1971). Another consequence of this equation is the prediction that a given overall surface area will cost least if distributed as unevenly as possible over a given number of separate exchangers. Compare, for example, two exchangers, both of surface area a, with two exchangers having surface areas $a + b$ and $a - b$, respectively:

$$2a^{0.6} > (a + b)^{0.6} + (a - b)^{0.6} \qquad (4)$$

This leads to a situation where optimum cost networks are proposed which will show maximum possible variation in exchanger size. In practice, the optimum variation of exchanger sizes will be less since the installed cost of small units tends to a constant value. In the present context, however, it was thought desirable to make strict comparisons between the results obtained here and those found by previous workers. For this reason, optimum cost solutions were synthesized according to the original equation.

* Their own pricing is $38 713/yr for their own solution and $38 762/yr for the single stream optimum structure. Nishida et al. used 260 hr/yr equipment downtime and 1 785.11 kJ/kg (767.5 Btu/lb) as latent heat of steam, compared to 380 hr/yr and 1 786.27 kJ/kg (768 Btu/lb) used by Pho and Lapidus (1973), McGalliard and Westerberg (1972), and this paper.

6SP1, 7SP2, AND 10SP1, FURTHER DEMONSTRATIONS OF THE ED METHOD

6SP1 was first presented by Lee et al. (1970). They found a solution which they claimed costs $35 110. However, as pointed out by Nishida et al. (1977), this figure is based on the same type of error as that of Pho and Lapidus in the case of 7SP1 (see above). The true cost of Lee et al.'s solution is $37 331/yr. Hohmann (1971) found a solution costing $35 010; McGalliard and Westerberg (1972) found one costing $35 780/yr, and Pho and Lapidus (1973) found one costing $35 657/yr. Nishida et al. showed the same solution as Hohmann. In this work, Hohmann's solution was confirmed as the optimum, see Figure 15*a*. With the TI and the ED methods, the network was found by deliberately creating a very small exchanger (that is, match No. 4) and, at the same time, by using the minimum number of units. In other words, a search through the solution space was found unnecessary to find this optimum cost network since the two features which would result in low cost (that is, minimum number of units together with extreme differences in exchanger size) were easily developed by means of the ED method. Another network, found by means of the same strategy, exhibits an annual cost of $35 017/yr (see Linnhoff, 1978).

However, both these networks feature differences in exchanger sizes which may not be practical; for the optimum cost structure, the ratio of largest transfer area to smallest transfer area is about 440. For this reason, an alternative solution for 6SP1 is shown in Figure 15*b*. It is a flexible structure, incorporating one piece of equipment more than strictly necessary and exhibiting a ratio of largest to smallest transfer area of between 8.0 and 5.5. Over most of the range of variation, the cost is still within 2% of the optimum. For these reasons, structure 15*b* is considered to be quite an attractive solution for 6SP1. It has been found by shifting heaters and coolers in such a way as to create a certain amount of difference in exchanger sizes to reduce cost but with practical limits in mind.

Problem 7SP2 was presented by Masso and Rudd (1969) as one for which the combinatorial possibilities are overwhelming. The heat recovery situation is very easy, and there are many feasible structures which use the minimum number of units and achieve maximum energy recovery. Masso and Rudd presented a solution costing $28 628/yr. Pho and Lapidus claimed to have found a solution costing $28 518/yr, but the structure they presented is not feasible since at least two streams miss their target temperatures. In Figure 11*a*, a new optimum cost solution for 7SP2 is shown which costs $28 258/yr. It was found by means of the same strategy as described above.

The most complex example from the literature is 10SP1. It was presented by Pho and Lapidus as a problem which would make it necessary, when using conventional synthesis algorithms, to approach the practical limits which exist for computer store and time. They found a solution costing $44 158/yr. Nishida et al. (1977) found a solution costing $43 984/yr. Figure 16*b* shows a solution costing $43 934/yr. This new optimum was found by means of the same strategy as employed when finding the optimum cost solutions for 6SP1 and 7SP2. It can, incidentally, be evolved in only a few steps from Pho and Lapidus's solution.

Practical Solution by Relaxing Problem Specifications

In Figure 17, a solution for 10SP1 is shown in which stream No. 3 misses its target temperature by 1.53°C. Owing to this slight relaxation of the problem data, it has been possible to design a network which consists of only nine pieces of equipment. (This is one less than the minimum number of units which would normally be necessary; the heat loads of streams No. 9 and No. 3 have, owing to the slight change in target temperature, become equal.) From a theoretical point of view, this solution is not of strictly optimal cost because the failure to recover 12.9 kW of heat results in an increase in the amount of cooling water required. However, the structure is less complex than any one of the correct solutions, and this will almost certainly mean lower costs for piping. In an industrial situation, structure 17 would probably warrant further investigation.

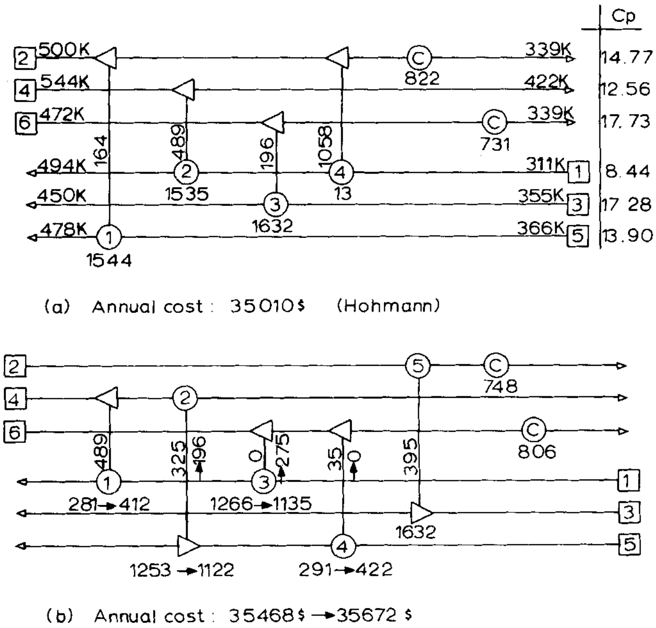

Fig. 15. Two solutions for 6SP1. (a) Optimum cost solution. (b) Flexible structure of near optimum cost.

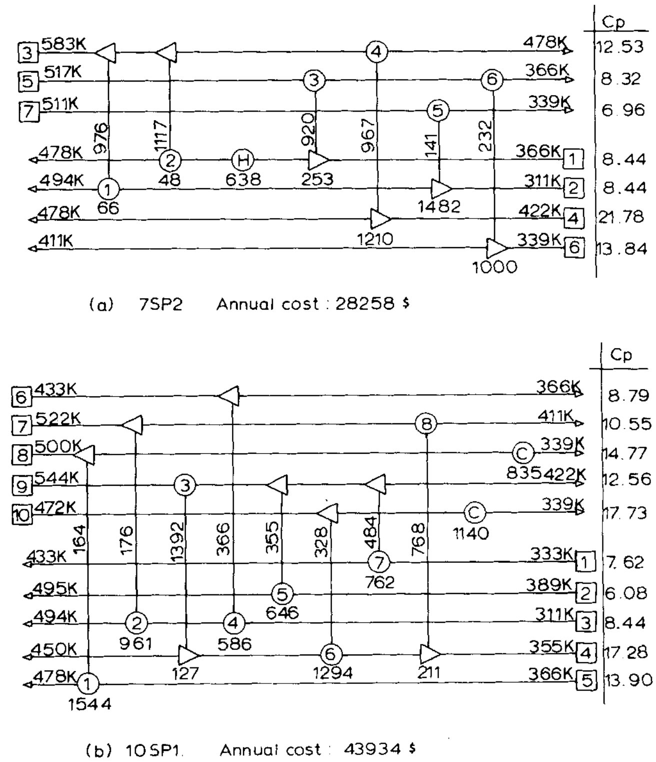

Fig. 16. Two new optimum cost solutions. (a) For 7SP2. (b) For 10SP1.

DISCUSSION

Cyclic Topologies and Parallel Stream Splitting

With the exception of 4SP2, all examples dealt with in this paper have been solved to give maximum energy recovery using the minimum number of units without the introduction of cyclic structures or parallel stream splitting. In the case of 4SP2, a cyclic topology was found necessary to achieve maximum energy recovery (see, for example, Figure 9*d*), while parallel stream splitting was found necessary to reduce the number of units to the minimum number (see, for example, Figure 10*c*). *Both techniques, the use of more exchangers than* suggested by expression (3) in an unsplit network, as well as parallel splitting of streams were seen to be powerful means of securing maximum energy recovery. Beyond this, the parallel splitting of streams sometimes also *allows the use of the minimum number of units at the* same time. There is no guarantee, however, that stream splitting solutions will always enable the minimum number of units to be realized (Linnhoff, 1978).

Stream Splitting and the Minimum Number of Units

As Ponton and Donaldson (1974) and Nishida et al. (1977) pointed out, the main difficulty in problem 4SP2 is the fact that the heat capacity flow rate of the cold stream is excessively large as compared to the three hot streams. This makes it impossible to recover all heat in a simple acyclic network. On the other hand, it is precisely the fact that stream No. 1 is excessively large, which makes such simple stream splitting solutions possible as shown in Figure 10. When evolving these structures, minimum figures were identified for the heat capacity flow rates of the different branches of stream No. 1 [see Equation (2)], and it is only due to the fact that the actual Cp of stream No. 1 exceeds the sum of these minimum figures that solutions 10*b* and 10*c* are feasible.

In the two test cases of Part I, a large and a small hot stream are to be exchanged with two cold streams of intermediate heat capacity flow rates. This situation may give rise to thermodynamic difficulties just as great as the rather obvious situation found in 4SP2, and, when coupled with unfavorable temperatures, simple structures will not allow maximum energy recovery (see solutions for test case No. 2 in Part I). As documented elsewhere (Linnhoff, 1978), the thermodynamic constraints in the two test cases are such that even stream splitting solutions have to consist of at least six pieces of equipment [that is, one more than the minimum according to expression (3)].

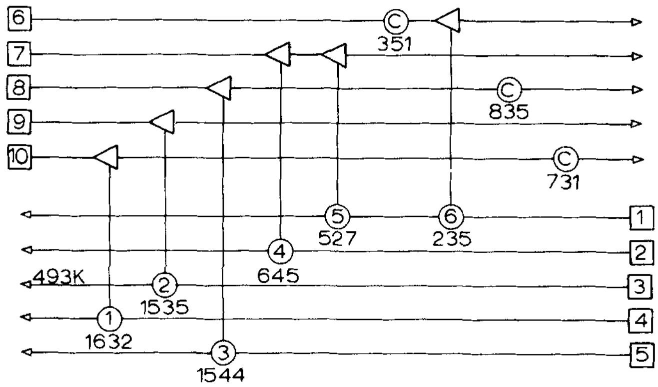

Fig. 17. Practical solution for 10SP1.

Stream Splitting and Flexibility

Last, but not least, the potential benefits of cyclic arrangements or stream splitting should be discussed even where maximum energy recovery can be obtained by simple unsplit solutions.

The introduction of stream splitting into any such structure can be considered by means of Figure 18. Nishida et al.'s optimum cost solution for 5SP1 is shown in Figure 18*a*. Figure 18*b*, taken from Figure 14*a*, shows that the only difference between the two structures is, in fact, the arrangement of exchangers. In Figure 18*b*, the two temperatures which will always show whether the introduction of stream splitting is possible are shown boxed; if the hot stream exit temperature of the second match is higher (by at least ΔT_{min}) than the cold stream entry temperature of the first match, the transition from the single stream to the split arrangement is definitely feasible. (An equivalent argument is easily formulated for the case of two matches which are connected in series to a cold stream.)

However, the transition may be feasible even if the two temperatures in question do not satisfy this test.

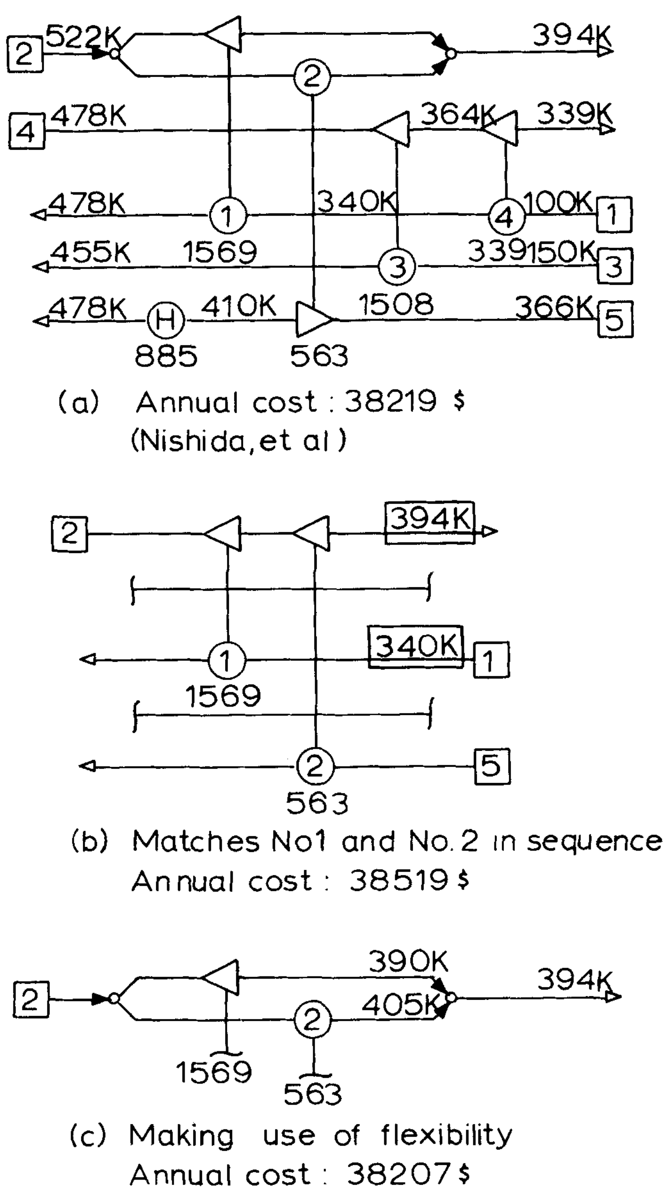

Fig. 18. Stream splitting for flexibility. (a) Nishida, et al.'s optimum cost solution for 5SP1. (b) Taken from Figure 14a. (c) Choice of heat capacity flowrates in the branches of stream No. 1 allows varying exchanger sizes.

After all, once one has introduced stream splitting, there is flexibility in the choice of one of the heat capacity flow rates of the newly produced branches of the split stream; the temperatures before the mixing point in Figure 18*a* can thus be adjusted appropriately. This extra degree of choice is not considered by Nishida et al. when proposing structure 18*a*. In Figure 18*c*, different values are given for the temperatures in question (arising from a different choice of heat capacity flow rates). The result is a marginal improvement in cost.

The important aspect of Figure 18*c*, however, is not the fact that a new optimum has been found but rather that an alternative way has been identified of introducing flexibility into network design. From a comparison of matches No. 1 and No. 2 in Figures 18*a* and 18*b*, it is clear that substantial differences in cost are unlikely to occur. The number of units remains unchanged and the changes in surface area will always be beneficial for one of the exchangers and detrimental for the other. However, in terms of flexibility, there is a distinct advantage. The surface areas of the two matches may be varied, within a certain range, just as in the case of flexible solutions produced by introducing additional items of equipment into a network. The interesting difference is the fact that the number of units need not be increased if flexibility is introduced by means of stream splitting. It appears that even in cases where stream splitting is not required to obtain maximum energy recovery, its main attraction might be in promoting flexibility. The feasible range over which parameters may be varied is easily established by an appropriate application of Equation (2).

The ED Method and Heuristic Based Evolutionary Rules

It is, perhaps, worth stressing once more that the ED method as introduced in this paper represents a means by which heuristic rules and strategies for the evolutionary improvement of heat exchanger networks may be implemented, but that it is itself based on elementary thermodynamics, that is, is of general validity. Although a number of apparently suitable heuristic strategies were formulated in the course of solving the various examples, the main contribution of this work is thought to be the fact that a theoretically based method has been found which greatly helps the rapid formulation of whatever strategy is suitable for a given type of synthesis task. In other words, the potential power which appears to be offered by the ED method to obtain increased insight into particular future problems might be more valuable an asset to the design engineer than heuristic rules based on limited past experience or on simplified cost equations.

Accordingly, no attempts have been made in this work to try, for example, to formulate detailed evolutionary rules which would invariably lead to a reduction of the number of units until the minimum number for any given network is reached. Such rules have been given by Nishida, et al. (1977), and it has been conceded that they might, sometimes, fail. With the ED method, it is easy to see why they will work in most cases, but it is just as easy to see when and why they will fail. What is more important is that if they fail, it is generally still easy to see by what other means the number of units may be reduced. (Compare Nishida et al.'s solution for 4SP2, that is, structure 7*b* to structure 10*c*.) Similarly, the same authors have given theoretical guidance as to where in a network utility heaters and coolers ought to be placed. With the ED method, such guidance is not required from a thermodynamic point of view.

On the other hand, the findings discussed in this work with regard to number of units, flexibility of design, stream splitting, and cyclic structures are probably of quite general relevance, and, based on these findings, the following recommendations are given.

General Recommendations

The following might be a promising approach to heat exchanger network design:

1. Identify the minimum number of units from expression (3), using utilities predicted by the problem table.
2. Synthesize a maximum energy recovery network by means of TI method and reduce number of units by means of ED method. (If thermodynamic constraints exist which make it difficult to approach the minimum number of units, stream splitting should be considered).
3. When the network is sufficiently close to a minimum number of units, promote features of individual interest and suppress unwanted characteristics.
4. With present pricing parameters or those used here for comparability with past work, maximum energy recovery is almost certain to be a feature of interest. However, different pricing parameters might make it worthwhile examining structures which offer lower capital costs with less than maximum energy recovery. In such a case, the strategy adopted should be to first examine those structures, if they exist, which feature fewer units than are found necessary to achieve optimum energy recovery. In other words, the guideline of the final stages of the synthesis should be the number of units rather than the degree of departure from maximum energy recovery. As an example of this strategy, see the solutions to test case No. 2 in Part I.

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Bodo Linnhoff gratefully acknowledges the financial support from The British Council which has made this work possible. The authors also wish to acknowledge the helpful comments of Dr. J. J. Siirola.

NOTATION

All numerical values quoted are based on calculations using original data in Imperial units. Inevitable rounding on conversion to S.I. units may lead to small discrepancies. Original data can be supplied by the authors.

A = heat load of a cooler or a heater to be shifted or merged, kW
a, b = given sizes of heat transfer surface area, m^2
Cp = heat capacity flow rate, kW/°K
CPL = larger Cp of the two streams matched in an exchanger, kW/°K
CPS = smaller Cp of the two streams matched in an exchanger, kW/°K
F = freedom of a heat exchanger, kW
n = number of separate exchangers
$SN(K)$ = K^{th} subnetwork
T_S = supply temperature, °K
T_T = target temperature, °K
z = number of streams in a problem
ΔT_{min} = minimum temperature difference within heat exchangers, °K
ΔTS = smallest actual temperature difference within heat exchangers, °K

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On-Line Gain Identification of Flow Processes with Application to Adaptive pH Control

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A simple and practical method is presented for the control of first-order flow processes with time varying gain and a pure delay in the measurement of the control variable. It involves on-line identification of the process gain and a subsequent adjustment of the controller parameter. The method is well suited for applications in the chemical process industries where high frequency fluctuations in the process gain are not generally expected. It is tested on a computer simulated process as well as experimentally by application to a continuous stirred-tank neutralization process involving *p*H control, where buffer concentration in the feed varies with time. There is a potential for the application of this method to an industrial process such as wastewater treatment.

SCOPE

Processes whose gains vary widely during operation cannot be controlled satisfactorily by conventional fixed-parameter controllers. A solution to such a problem is to identify the process gain on-line and adjust the controller parameter suitably. This paper presents a method of on-line gain identification and control that applies to first-order flow processes with time-varying gain. An identification system (also of first order) in series with the process being controlled is perturbed by a signal consisting of a variable frequency, constant amplitude, rectangular wave to generate discrete estimates of the process gain. The identification is based on the fact that for a constant input to the identification system, the time taken for the identification system output to go from one preselected value to another is inversely proportional to the gain. The gain identification is achieved without disturbing the normal operation of the plant. The method is simpler and more easily implemented than the sinusoidal perturbation method proposed by Mellichamp et al. (1966).

The control strategy is based on maintaining a constant loop gain so as to maintain the same degree of stability. Thus, employing a two-mode (PI) or three-mode (PID) controller, the controller gain is self-adjusted as the process gain varies such that their product remains constant. All on-line computation, data storage, and control are handled by a moderate size analog computer with patchable logic.

The proposed method of identification and control is first tested on a computer-simulated process and then applied experimentally to a *p*H control system involving continuous stirred-tank neutralization of an acidic stream containing monobasic phosphoric acid as the buffer species by potassium hydroxide. This system can be modeled as a first-order process with time-varying gain. The variation in the process gain is caused by a change in the concentration of the buffer species in the feed stream. The perturbation input to the identification system consists of alternate flow of an acid (nitric acid) and a base (potassium hydroxide) at constant flow rate.

Process gain changes as large as 10 to 1 are introduced.

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